

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS (SYLLABUS D)

4024/02
Paper 2

## October/November 2008

2 hours 30 minutes
Additional Materials: Answer Booklet/Paper
Graph paper (2 sheets)
Electronic calculator Geometrical instruments

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100 .

## Section A [52 marks]

Answer all questions in this section.

1 (a) In 2006 the cost of fuel was 91.8 cents per litre.
(i) Calculate the maximum number of whole litres that could be bought for $\$ 15$.
(ii) In 2007 the cost of fuel was increased by 4 cents per litre.
(a) Calculate the percentage increase in the cost of fuel in 2007.
(b) On average, a car travelled 21 km on 1 litre of fuel.

During 2006 this car travelled 19200 km .
In 2007 the car travelled the same distance.
Calculate the extra cost for fuel in 2007.
Give your answer to the nearest dollar.
(iii) In 2006 the cost of fuel was $10 \%$ less than the cost in 2005.

Calculate the cost, in cents, per litre in 2005.
(b) A service station sells unleaded, super-unleaded and diesel fuel.

The pie chart represents the amounts of these fuels sold during one week.


The total amount of fuel sold during this week was 54000 litres.
(i) How many litres of diesel were sold?
(ii) The amount of unleaded fuel sold was $\frac{2}{3}$ of the total for the week.

How many litres of super-unleaded fuel were sold?

2 Eight straight paths in a level garden form this shape with rotational symmetry of order four.

(a)


The two paths shown, $A B$ and $B C$, form part of the isosceles triangle $A B C$.
$A C=10 \mathrm{~m}$ and angle $B A C=65^{\circ}$.
Calculate
(i) the length of the path $A B$,
(ii) the area of triangle $A B C$,
(iii) the area of garden enclosed by all 8 paths.
(b)


Ada walked along the paths $B C$ and $C D$.
(i) Calculate $B \hat{C} D$.
(ii) After walking in the direction $B C$, Ada turned to walk in the direction $C D$.

State the value of the angle through which she turned.

3
(a) Solve the equation

$$
\begin{equation*}
\frac{2 p+1}{3}=1+\frac{p-3}{2} . \tag{3}
\end{equation*}
$$

(b) Simplify

$$
\begin{equation*}
\frac{2 v-6}{v^{2}-2 v-3} \tag{3}
\end{equation*}
$$

(c) The tens digit of a number is $x$ and the units digit is $y$.

Hence the value of the number is $10 x+y$.
For example, if $x=5$ and $y=6$, the number would be $10 \times 5+6=56$.
(i) When the digits $x$ and $y$ are reversed, the value of the number is increased by 63 .

Show that $y-x=7$.
(ii) The sum of the original number and the number with reversed digits is 99 .
(a) Show that $x+y=9$.
(b) Hence find the value of $x$ and the value of $y$.

## 4 Answer the whole of this question on a sheet of graph paper.

The heights of 120 children were measured.
The results are summarised in the table below.

| Height $(h \mathrm{~cm})$ | $135<h \leqslant 140$ | $140<h \leqslant 145$ | $145<h \leqslant 150$ | $150<h \leqslant 155$ | $155<h \leqslant 160$ | $160<h \leqslant 180$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 15 | 20 | 25 | 30 | 20 | 10 |

(a) Using a scale of 1 cm to represent 5 cm , draw a horizontal axis for heights from 135 cm to 180 cm . Using a scale of 2 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 6 units.
On your axes, draw a histogram to represent the information in the table.
(b) Estimate how many children have heights greater than 170 cm .
(c) One child was chosen at random.

Find the probability that the height of this child was less than or equal to 140 cm .
Give your answer as a fraction in its lowest terms.
(d) Two children were chosen at random.

Find the probability that they both had heights in the range $150<h \leqslant 155$.


The diagram shows a circle, centre $O$.
The lines $A B, B C$ and $C A$ touch the circle at $P, Q$ and $R$ respectively.
(a) (i) Explain why $C \hat{Q} O=90^{\circ}$.
(ii) Given that $A \hat{C} B=40^{\circ}$, find $R \hat{O} Q$.
(b) The line $D E$ touches the circle at $S$.

The triangles $A B C$ and $A D E$ are similar.
(i) Write down the value of $A \hat{E} D$.
(ii) Given that $R \hat{O} P=100^{\circ}$, find $R \hat{O} S$.
(iii)


Given also that $A D=3 \mathrm{~cm}, C D=17 \mathrm{~cm}$ and $A E=4 \mathrm{~cm}$, calculate $B E$.

6 (a) Each diagram in the sequence below consists of a number of dots.

Diagram number 1
$1 \quad 2$

3
4
Number of dots
$T_{1}$
$T_{2}$
$T_{3}$
$T_{4}$

| Diagram number $(n)$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of dots $\left(T_{n}\right)$ | 1 | 5 | 11 | $p$ | $q$ |

(i) Write down the value of $p$.
(ii) Find the value of $q$.
(b) Another sequence of patterns of dots is shown below.


| Diagram number $(n)$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of dots $\left(S_{n}\right)$ | 1 | 4 | 9 | $j$ | $k$ |

(i) Write down the value of $j$.
(ii) Find the value of $k$.
(iii) Express $S_{n}$ in terms of $n$.
(c) Another sequence is formed whose $n$th term $U_{n}$ is $T_{n}-S_{n}$.
$U_{1}=T_{1}-S_{1}=1-1=0, \quad U_{2}=T_{2}-S_{2}=5-4=1, \quad U_{3}=T_{3}-S_{3}=11-9=2$,
(i) Evaluate $U_{4}$ and $U_{5}$.
(ii) Express $U_{n}$ in terms of $n$.
(iii) Hence, using your expression for $S_{n}$ found in (b)(iii), express $T_{n}$ in terms of $n$.

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

7 A light aircraft flew from Maseru to Nata and returned to Maseru.
(a) The distance from Maseru to Nata is 1080 km .
(i) On the outward flight, the average speed of the aircraft was $x$ kilometres per hour. Write down an expression, in terms of $x$, for the time taken in hours.
(ii) On the return flight, the average speed was $30 \mathrm{~km} / \mathrm{h}$ greater than the average speed on the outward flight.
Write down an expression, in terms of $x$, for the time taken, in hours, on the return flight. [1]
(b) The time taken on the return flight was half an hour less than the time taken on the outward flight. Form an equation in $x$ and show that it reduces to $x^{2}+30 x-64800=0$.
(c) Solve the equation $x^{2}+30 x-64800=0$.
(d) Calculate
(i) the time taken, in hours, on the outward flight,
(ii) the average speed for the whole flight from Maseru to Nata and back to Maseru.

$C$ is a point on a slope with highest point $B$, where $C B=10 \mathrm{~m}$.
A vertical mast, $A B$, of height 15 m stands at $B$ and $A \hat{B} C=105^{\circ}$.
$A C$ is one of the wires that supports the mast.
(a) (i) Write down the angle between $C B$ and the horizontal.
(ii) Calculate the length of the wire $A C$.
(b)

$D$ is another point further down the slope.
$A D$ is another supporting wire.
Given that $A D=30 \mathrm{~m}$, calculate $A \hat{D} B$.
(c)


The points $B, E$ and $F$ are on horizontal ground at the top of the slope.
The mast is supported by another wire $A F$.
$B \hat{E} F=90^{\circ}, A F=27 \mathrm{~m}$ and $B E=20 \mathrm{~m}$.
Calculate
(i) $E F$,
(ii) the angle of elevation of the top of the mast from $F$.

9 (a) The diagram shows a roll of material. The material is wound onto a metal cylinder whose cross-section is a circle of radius 10 cm .
The shaded area shows the cross-section of the material on the roll. The outer layer of material forms the curved surface of a cylinder of radius 30 cm .

(i) Calculate, in square centimetres, the area of the cross-section of the material on the roll (shaded on the diagram).
(ii) The material is 200 cm wide on the roll.

Calculate, in cubic metres, the volume of the material.
(iii) When unwound, the length of the material is 150 m .

Calculate the thickness of the material, giving your answer in millimetres.
(b) The diagram shows a conical tent.

The diameter of the base is 3.5 m and the slant height is 3 m .
It is made from a flat piece of canvas that forms a sector of a circle of radius 3 m . The angle at the centre is $\theta^{\circ}$.

(i) Show that $\theta=210$.
(ii) As shown, the required shape is cut from a rectangular piece of canvas of width $w$ metres.


Given that $w$ is a whole number, find its least possible value. Show all your working.

## 10 Answer the whole of this question on a sheet of graph paper.

The number of bacteria in a colony doubles every half hour.
The colony starts with 50 bacteria.
The table below shows the number of bacteria in the colony after $t$ hours.

| Time $(t$ hours $)$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of bacteria $(y)$ | 50 | 100 | 200 | 400 | 800 | 1600 | 3200 | 6400 |

(a) Using a scale of 4 cm to represent 1 hour, draw a horizontal $t$-axis for $0 \leqslant t \leqslant 4$.

Using a scale of 2 cm to represent 1000 bacteria, draw a vertical $y$-axis for $0 \leqslant y \leqslant 7000$.
On your axes, plot the points given in the table, and join them with a smooth curve.
(b) Use your graph to find the number of bacteria in the colony when $t=2.75$.
(c) (i) By drawing a tangent, find the gradient of the graph when $t=2.5$.
(ii) State briefly what this gradient represents.
(d) The number of bacteria in another colony is given by the equation

$$
y=6500-1000 t .
$$

(i) On the same axes, draw a graph to represent the number of bacteria in this colony.
(ii) Find the value of $t$ when the number of bacteria in each colony is the same.
(e) Given that the equation of the first graph is $y=k a^{t}$, find the value of
(i) $k$,
(ii) $a$.

11 (a) $\overrightarrow{P Q}=\binom{12}{-35}$ and $\overrightarrow{Q R}=\binom{4}{14}$.
(i) Find
(a) $|\overrightarrow{P Q}|$,
(b) $\overrightarrow{P R}$.
(ii) Given that $T$ is the midpoint of $Q R$, find $\overrightarrow{P T}$.
$\underbrace{P}_{Q}$
(iii) $P Q R S$ is a parallelogram.

The coordinates of $R$ are $(6,16)$.
Find the coordinates of $S$.
(b)


The diagram shows triangle $A B C$.
(i) Find the area of triangle $A B C$.
(ii) An enlargement, scale factor 4, maps triangle $A B C$ onto triangle $L M N$.

The point $A$ maps onto the point $L(10,3)$.
(a) Find the coordinates of the centre of enlargement.
(b) Write down the area of triangle $L M N$.
(iii) A shear, with the $x$-axis invariant, maps triangle $A B C$ onto triangle $D E F$.

The point $A$ maps onto the point $D(7,3)$.
(a) Find the coordinates of $E$, the image of $B$.
(b) Write down the area of triangle $D E F$.

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